

# DefElement

an encyclopedia of finite element definitions

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## What is DefElement?

DefElement is an online encyclopedia of finite element definitions. You can view it at

[defelement.com](https://defelement.com)

DefElement includes definitions of a huge range of finite elements including commonly used elements such as Lagrange, Raviart–Thomas [6], and Nédélec [4, 5]; and more exotic elements such as Argyris [1], Regge [7, 2], and TNT [3].

## What information is on DefElement?

- Name(s) of the element
- Definition and properties of the element
- Implementations of the element
- Example DOF diagrams and basis functions, with plots created using Symfem [8]
- References

All the information and diagrams on DefElement are available for reuse under a Creative Commons CC BY 4.0 license: you can use them for free as long as you link to or cite DefElement. All the diagrams are available to download in PNG, SVG, and TikZ formats.

## Can I contribute to DefElement?

Yes! DefElement's source code is available on GitHub (MIT license). You can contribute by opening GitHub issues for:

- New elements that could be added to DefElement.
- Any improvements that you want to suggest.
- Any mistakes that you find.

Or, you could fork the repository and open a pull request to:

- Add implementation information for a finite element library that you use or maintain.
- Resolve any of the currently open issues: keep an eye out for anything tagged *good first issue*.
- Anything else you want to suggest changing.

**Raviart–Thomas**

[Click here to read what the information on this page means.](#)

ALTERNATIVE NAMES	Rao–Wilton–Glisson, Nédélec (first kind) H(div)
DE RHAM COMPLEX FAMILIES	$[S_{2,k}^d]_{d=1}^3 / \mathcal{P}_k^d \Lambda^{d-1}(\Delta_d)$
ABBREVIATED NAMES	RT, RWG
ORDERS	$1 \leq k$
REFERENCE ELEMENTS	<a href="#">triangle</a> , <a href="#">tetrahedron</a>
POLYNOMIAL SET	$\mathcal{P}_{k-1}^d \oplus \mathcal{Z}_k^{(25)}$ <a href="#">Show polynomial set definitions ↓</a>
DOFS	On each facet: normal integral moments with an order $k-1$ <a href="#">Lagrange</a> space On the interior of the reference element: integral moments with an order $k-2$ <a href="#">vector Lagrange</a> space
NUMBER OF DOFS	triangle: $k(k+2)$ ( <a href="#">A005563</a> ) tetrahedron: $k(k+1)(k+3)/2$ ( <a href="#">A077414</a> )
MAPPING	contravariant Piola
CONTINUITY	Components normal to facets are continuous
CATEGORIES	<a href="#">Vector-valued elements</a> , <a href="#">H(div) conforming elements</a>

**Implementations**

BASIX	basix.ElementFamily.RT <a href="#">Show Basix examples ↓</a>
BEMPP	"RWG" (triangle) <a href="#">Show Bempp examples ↓</a>
SYMFEM	"Nidiv" <a href="#">Show Symfem examples ↓</a>
UFL	"RT" <a href="#">Show UFL examples ↓</a>

**Examples**

TRIANGLE ORDER 1	 <a href="#">(click to view basis functions)</a>
TRIANGLE ORDER 2	 <a href="#">(click to view basis functions)</a>
TETRAHEDRON ORDER 1	 <a href="#">(click to view basis functions)</a>
TETRAHEDRON ORDER 2	 <a href="#">(click to view basis functions)</a>

**References**

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**DefElement stats**

ELEMENT ADDED	30 December 2020
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## References

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[2] Snorre H. Christiansen. On the linearization of Regge calculus. *Numerische Mathematik*, 119(4):613–640, 2011.

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[6] Pierre-Arnaud Raviart and Jean-Marie Thomas. A mixed finite element method for 2nd order elliptic problems. In Ilio Galligani and Enrico Magenes, editors, *Mathematical aspects of finite element methods*, volume 606, pages 292–315, 1977.

[7] Tullio Regge. General relativity without coordinates. *Il Nuovo Cimento*, 19(3):558–571, 1961.

[8] Matthew W. Scroggs. Symfem: a symbolic finite element definition library. *Journal of Open Source Software*, 6(6):3556, 2021.