

# DefElement

## an encyclopedia of finite element definitions

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### What is DefElement?

DefElement is an online encyclopedia of finite element definitions. You can view it at

[defelement.com](https://defelement.com)

DefElement includes definitions of a huge range of finite elements including commonly used elements such as Lagrange, Raviart–Thomas [6], and Nédélec [4, 5]; and more exotic elements such as Argyris [1], Regge [7, 2], and TNT [3].

### What information is on DefElement?

- Name(s) of the element
- Definition and properties of the element
- Implementations of the element
- Example DOF diagrams and basis functions, with plots created using Symfem [8]
- References

All the information and diagrams on DefElement are available for reuse under a Creative Commons CC BY 4.0 license: you can use them for free as long as you link to or cite DefElement. All the diagrams are available to download in PNG, SVG, and TikZ formats.

### Can I contribute to DefElement?

Yes! DefElement's source code is available on GitHub (MIT license). You can contribute by opening GitHub issues for:

- New elements that could be added to DefElement.
- Any improvements that you want to suggest.
- Any mistakes that you find.

Or, you could fork the repository and open a pull request to:

- Add implementation information for a finite element library that you use or maintain.
- Resolve any of the currently open issues: keep an eye out for anything tagged *good first issue*.
- Anything else you want to suggest changing.

**Raviart–Thomas**

Click here to read what the information on this page means.

ALTERNATIVE NAMES	Rao–Wilton–Glisson, Nédélec (first kind) H(div)
DE RHAM COMPLEX FAMILIES	$[S_{2,k}]_{d-1} / \mathcal{P}_k A^{d-1}(\Delta_d)$
ABBREVIATED NAMES	RT, RWG
ORDERS	$1 \leq k$
REFERENCE ELEMENTS	triangle, tetrahedron
POLYNOMIAL SET	$\mathcal{P}_{k-1}^d \oplus \mathcal{Z}_k^{(25)}$ <a href="#">Show polynomial set definitions ↓</a>
DOFS	On each facet: normal integral moments with an order $k - 1$ Lagrange space On the interior of the reference element: integral moments with an order $k - 2$ vector Lagrange space
NUMBER OF DOFS	triangle: $k(k + 2)$ (A005563) tetrahedron: $k(k + 1)(k + 3)/2$ (A077414)
MAPPING	contravariant Piola
CONTINUITY	Components normal to facets are continuous
CATEGORIES	<a href="#">Vector-valued elements</a> , <a href="#">H(div) conforming elements</a>

**Implementations**

basix.ElementFamily.RT  
[Show Basix examples ↓](#)

BEMPP  
"RWG" (triangle)  
[Show Bempp examples ↓](#)

SYMFEM  
"Nidiv"  
[Show Symfem examples ↓](#)

UFL  
"RT"  
[Show UFL examples ↓](#)

**Examples**

TRIANGLE ORDER 1  
  
[\(click to view basis functions\)](#)

TRIANGLE ORDER 2  
  
[\(click to view basis functions\)](#)

TETRAHEDRON ORDER 1  
  
[\(click to view basis functions\)](#)

TETRAHEDRON ORDER 2  
  
[\(click to view basis functions\)](#)

**References**

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**DefElement stats**

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#### References

[1] John H. Argyris, Isaac Fried, and Dieter W. Scharpf. The TUBA Family of Plate Elements for the Matrix Displacement Method. *The Aeronautical Journal*, 72(692):701–709, 1968.

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[7] Tullio Regge. General relativity without coordinates. *il Nuovo Cimento*, 19(3):558–571, 1961.

[8] Matthew W. Scroggs. Symfem: a symbolic finite element definition library. *Journal of Open Source Software*, 6(4):3556, 2021.